MATH 54 - HINTS TO HOMEWORK 8

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Here are a couple of hints to Homework 8! Enjoy! :)

Note: Exam-wise, section 6.7 is the most important section, so make sure to study it in detail. However, there might be problems from 6.6 and 6.8 that might be graded!

Section 6.6: Applications to linear models

6.6.1, 6.6.3. Use equation (1) on page 360. It's basically just solving a least-squares problem with a special matrix A.

6.6.7.

(a)

$$X = \begin{bmatrix} x_1 & x_1^2 \\ x_2 & x_2^2 \\ x_3 & x_3^2 \\ x_4 & x_4^2 \\ x_5 & x_5^2 \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

where $(x_1, y_1) = (1, 1.8)$ etc.

(b) You can ignore this part if you want! (or use MATLAB)

6.6.13. OH GOD! IGNORE, IGNORE, IGNORE!!! You have better things to do! :)

6.6.15. Start with $X^T X\hat{\beta} = X^T\mathbf{y}$.

Then, by definition of X, we get:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

That is:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 + \hat{\beta}_1 x_1 \\ \hat{\beta}_0 + \hat{\beta}_1 x_2 \\ \vdots \\ \hat{\beta}_1 x_n \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

NOW expand the left-hand-side and the right-hand-side out, and what you should get are the normal equations, using the notation $\sum x$ and $\sum y$ etc. introduced right before the exercise!

Date: Friday, October 14th, 2011.

Section 6.7: Inner product spaces

6.7.1. Here $\langle \mathbf{x}, \mathbf{y} \rangle = 4u_1v_1 + 5u_2v_2$

6.7.3, 6.7.5, 6.7.7. Here $\langle p, q \rangle = p(-1)q(-1)+p(0)q(0)+p(1)q(1)$. And $\|p\| = \sqrt{\langle p, p \rangle}$. Finally, remember that the formula for orthogonal projection remains the same, namely:

$$\hat{q} = \frac{\langle q, p \rangle}{\langle p, p \rangle} p$$

6.7.9. Here $\langle p, q \rangle = p(-3)q(-3) + p(-1)q(-1) + p(1)q(1) + p(3)q(3)$

(a)

$$\hat{p_2} = \frac{\langle p_2, p_0 \rangle}{\langle p_0, p_0 \rangle} p_0 + \frac{\langle p_2, p_1 \rangle}{\langle p_1, p_1 \rangle} p_1$$

(b) $q = p_2 - \hat{p_2}$! (as usual! :)).

For the second part, all that is says is that multiply q by a constant, so that the new polynomial cq satisfies $cq(-3) = 1$, $cq(-1) = -1$ etc.

6.7.11. Here $\langle p, q \rangle = p(-2)q(-2) + p(-1)q(-1) + p(0)q(0) + p(1)q(1) + p(2)q(2)$.

If we let $p_3 = t^2$, then we have:

$$\hat{p_3} = \frac{\langle p_3, p_0 \rangle}{\langle p_0, p_0 \rangle} p_0 + \frac{\langle p_3, p_1 \rangle}{\langle p_1, p_1 \rangle} p_1 + \frac{\langle p_3, p_2 \rangle}{\langle p_2, p_2 \rangle} p_2$$

6.7.13. Show that it satisfies axioms **1, 2, 3, 4** in the definition of an inner product space (page 368). You may want to use the fact that $(A+B)^T = A^T + B^T$ and $(cA)^T = cA^T$. Also, for **4**, remember that $\mathbf{w}^T \mathbf{w} = w \cdot w \geq 0$ unless $\mathbf{w} = \mathbf{0}$.

6.7.19. The Cauchy-Schwarz inequality says $|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \, \|\mathbf{v}\|$

6.7.22, 6.7.24. $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$. Also, $\|g\| = \sqrt{\langle g, g \rangle}$

Section 6.8: Applications of inner product spaces

6.8.1. This is the same as fitting data to a line (section 6.6), except you multiply the equation $A\mathbf{x} = \mathbf{y}$ on the left by the diagonal matrix W of the weights to obtain $WA\mathbf{x} = W\mathbf{y}$. For example, in the first problem, we have:

$$W = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Now solve the equation $WA\mathbf{x} = W\mathbf{y}$ by using least-squares!

6.8.2. Should give you the same result, because solving $WA\mathbf{x} = W\mathbf{y}$ is the same as solving $(2W)A\mathbf{x} = (2W)\mathbf{y}$.

6.8.3. Use the formula in example 2 (on page 380), except you have to add the term $\frac{\langle g, p_3 \rangle}{\langle p_3, p_3 \rangle} p_3$

6.8.4.

(a) Use the Gram-Schmidt process on the set $\mathcal{B} = \left\{1, t, t^2\right\}$ with respect to the inner product

$$\langle p, q \rangle = p(-5)q(-5) + p(-3)q(-3) + p(-1)q(-1) + p(1)q(1) + p(3)q(3) + p(5)q(5)$$

(b) Just imitate example 2.